

# Assessing Robustness of Risk-Constrained Operating Strategies for Power Systems with Renewables by Contamination-Based Technique

Yujia Li

Department of Electrical and  
Electronic Engineering  
The University of Hong Kong  
Hong Kong, China  
yjli@eee.hku.hk

Shuanglei Feng

China Electric Power Research  
Institute  
Beijing, China  
fengsl@epri.sgcc.com.cn

Yunhe Hou

Department of Electrical and  
Electronic Engineering  
The University of Hong Kong  
Hong Kong, China  
yhhou@eee.hku.hk

**Abstract**—Risk-constrained stochastic programming (SP) is an effective tool to cope with the increasing uncertainty renewable energy resources (RESs) bring in look-ahead dispatch (LAD). However, since the solutions of SP are dependent on characteristics of uncertain RESs, inaccurate predication of RESs significantly influences the LAD strategies. This paper proposes a contamination-based technique (CBT) to evaluate the robustness of a dispatch strategy against inaccurate RESs predication. Without loss of generality, the proposed CBT is used in a two-stage CVaR-constrained stochastic program, where the first stage optimizes a strategy based on the original inaccurate predication and the second stage corrects this strategy by contaminating the first-stage inaccurate predication with updated information. The robustness of a strategy against the inaccurate predication is quantified by this method and, furthermore, the sensitivities of some critical parameters, such as penetration level of RESs and system flexibility, are analyzed. Case study validates the feasibility of proposed method.

**Index Terms**—contamination-based technique (CBT), wind uncertainty, robustness test, probability distribution function (PDF), Conditional Value at Risk (CVaR)

## NOMENCLATURE

### A. Sets and indices

$n/n_w/n_d$  Index for thermal units/wind turbines/load bus  
 $t$  Index for time period  
 $N_g/N_d/N_d$  Set for thermal units/wind turbines/load bus indices  
 $T$  Set for time period indices

### B. Parameters

$c_n^{g(1)}/c_n^{g(2)}$  Energy price for thermal unit  
 $\hat{c}_n^{g(1)}/\hat{c}_n^{g(2)}$  Up reserve cost for thermal unit at first/second stage  
 $\check{c}_n^{g(1)}/\check{c}_n^{g(2)}$  Down reserve cost for thermal unit at first/second stage

$\check{c}_n^{g(1)}/\check{c}_n^{g(2)}$  Down reserve cost for thermal unit at first/second stage  
 $c^{curt}$  Penalty cost for wind curtailment  
 $c^{shed}$  Penalty cost for wind curtailment  
 $\bar{d}_{n,d,t}$  Expected mean value of load  
 $\bar{w}_{n,w,t}(\cdot)$  Expected mean value of wind output  
 $ra_n$  Ramping rate of thermal unit  
 $fl^{min}/fl^{max}$  Power flow limit of transmission lines  
 $g_n^{min}/g_n^{max}$  Upper/lower output limit for thermal unit  
 $R_t^d/R_t^w$  Risk tolerance for load shedding and wind curtailment

### C. Decision Variables and Random Variables

$g_{n,t}/\Delta g_{n,t}$  Energy procured at first/second stage  
 $\hat{g}_{n,t}/\check{g}_{n,t}$  Up and down reserve procured at first stage  
 $\Delta \hat{g}_{n,t}/\Delta \check{g}_{n,t}$  Up and down reserve procured at second stage  
 $\tilde{w}_{n,w,t}/\Delta \tilde{w}_{n,w,t}$  Scheduled curtailed wind at first/second stage  
 $\check{d}_{n,d,t}$  Scheduled load shedding at second stage  
 $fl_t(\mathbf{0})/fl_t(\alpha)$  Vector of DC power flow at first/second stage  
 $\gamma_{s,t}^d/\gamma_{s,t}^w/v_t^d/v_t^w$  Auxiliary variables for CVaR calculation  
 $\mathbf{x}^{(1)}/\mathbf{x}^{(2)}$  Vector of decision variables at first/second stage  
 $\tilde{w}_{n,w,t}/\check{d}_{n,d,t}$  Random wind output/load level

## I. INTRODUCTION

Due to the merits of zero carbon emission and unlimited supply, renewable energy resources (RESs) have gathered great momentum all over the world. However, due to its inherently variable and uncertain nature, integrating large scale of nonsynchronous RESs into existing grid has posed great challenges to the system operation. Look ahead dispatch (LAD) associated with stochastic programming (SP) has been leveraged as one of standard techniques to cope with uncertainties and variabilities of RESs [1-3]. A day-ahead stochastic economic dispatch model is proposed in [1] to alleviate the uncertainties wind and demand response bring. In

[2], a mechanism, which incorporates the initial and revised decisions, was established. By this method, the improvement of the accuracies of RESs' predictions with the decision time approaching to the operating time point are modelled. Although numerous researches have been conducted based on the SP methodology, most of them are established based on a fundamentally critical assumption, i.e., the stochastic characteristics of RESs can be modelled by a fixed probability distribution function (PDF) or a set of scenarios [3].

Constructing operating strategy based on the fixed stochastic characteristics of RESs significantly challenges the current RESs prediction technology, especially for the mid-/long-term. Even though multiple literatures have evaluated the superiority of utilizing SP in LAD [3], their robustness against imprecise quantification of RESs' uncertainty have not been assessed adequately. Owing to the nature of SP, the feasible set and optimal value both have strong coupling with the characteristics of random variables. However, due to various reasons such as poor approximation, simplification or sudden disturbances, a fixed PDF or scenario set for random variables often has limited capability in representing the real pattern of RESs' outputs. For instance, Gaussian distribution is one of options to represent PDF of the wind prediction error, such as in [4], but it neglects many statistical properties by only considering variance and mean error, which distorts the "true" optimal dispatch cost and schedule [5]. Furthermore, some extreme events with high probability of occurrence is difficult to be presented in those fixed patterns. Thus in practical terms, due to growing penetration of RESs and increasing extreme events observed these years [6], we expect our decision made through SP are robust and reliable, even when PDF for RESs is approximated or suddenly changed.

Contamination technique, originated in mathematical statistics, is an effective tool to test the robustness of stochastic programs. Through perturbing the original PDF  $P$  by another fixed PDF  $Q$  continuously, the resistivity of stochastic model to the instability of PDF for random variable can be quantified and captured. It has been utilized in financial field to test the resilience of portfolio problem where PDFs for different assets cannot be described precisely [7]. In this paper, we propose a contamination based technique (CBT) to evaluate the robustness of an operating strategy established by SP where the prediction of RESs' output used in model is different from their real output patterns.

To this end, a contamination-based two-stage stochastic LAD model with CVaR constraints is formulated, where CBT is applied to evaluate the robustness of proposed stochastic model against RESs' uncertainty. Unlike conventional multi-stage models which update information by directly providing another determinate PDF, the proposed framework allows continuous perturbation on PDF by not giving it a fixed pattern. The result reveals that the proposed stochastic LAD model is relatively weak in withstanding the situation where wind is overvalued at the first stage, due to limited flexibility. Furthermore, while higher wind penetration level deteriorates the model robustness, increasing system flexibility can provide a positive effect.

## II. CONTAMINATION-BASED TECHNIQUE

The main idea and formulation of CBT are illustrated in this section. First we consider a risk-constrained stochastic program with following formulation:

$$\begin{cases} \varphi(P) = \min_{\mathbf{x} \in X} f(\mathbf{x}, P) \\ \text{s.t. } g_i(\mathbf{x}, P) \leq 0, i = 1, \dots, I \end{cases} \quad (1)$$

where  $\mathbf{x}$  denotes the vector of decision variables in compact form,  $P$  is a given PDF for a random vector  $\tilde{\mathbf{x}}$ ,  $X$  is a convex set for  $\mathbf{x}$  which is independent of  $P$ . Function  $\varphi(P)$ ,  $f(\mathbf{x}, P)$  and  $g_i(\mathbf{x}, P)$  ( $i = 1, \dots, I$ ) denote optimal value, objective function and constraints, respectively, all of which are dependent on  $P$ .

Then, we assume  $P$  cannot reflect the real pattern of  $\tilde{\mathbf{x}}$  precisely due to various reasons such as poor approximation and sudden external disturbances. This change can be presented by perturbing  $P$  with another fixed probability distribution  $Q$  continuously. The contaminated process can be written as follows:

$$P_\alpha = (1 - \alpha)P + \alpha Q, \alpha \in [0, 1] \quad (2)$$

where  $\alpha$  denotes the contamination degree, i.e., when  $\alpha = 0$ , the  $P$  remains uncontaminated and pure, and random vector  $\tilde{\mathbf{x}}$  perfectly follows the distribution  $P$ ; when  $\alpha = 1$ ,  $P$  is totally contaminated and random vector  $\tilde{\mathbf{x}}$  perfectly follows distribution  $Q$ ; when  $0 \leq \alpha \leq 1$ ,  $P$  is contaminated by  $Q$  partially with certain degree  $\alpha$ . Then (1) under the contamination can be rewritten as follows:

$$\begin{cases} \varphi(\alpha) = \min_{\mathbf{x} \in X} f(\mathbf{x}, \alpha) \\ \text{s.t. } g_i(\mathbf{x}, \alpha) \leq 0, i = 1, \dots, I \end{cases} \quad (3)$$

From the above formation (3), we can easily find that functions  $\varphi(\alpha)$ ,  $f(\mathbf{x}, \alpha)$  and  $g_i(\mathbf{x}, \alpha)$  are directly dependent on  $\alpha$ , as probability distribution  $P$  and  $Q$  are both determinate. Assume  $X(\alpha)$  denotes the feasible set for  $\mathbf{x}$  of problem (3), it obviously varies in accordance to  $\alpha$ , thus the optimizer  $X^*(\alpha)$  of (3) also varies with  $\alpha$ . Therefore, by setting different value of  $\alpha$ , we can evaluate the impact of adopting the distribution with different degree of inaccuracy.

## III. CONTAMINATION-BASED TWO-STAGE LAD MODEL

In this section, a contamination-based two-stage LAD model is proposed to evaluate the robustness of an operating strategy. At the first stage, energy and reserve are optimized based on the RES's output given by a PDF  $P_t^w(0)$ , which, physically, may be obtained by predication. After the first-stage decision has been already implemented, i.e., the decided capacity of energy and reserve have been already procured, a sudden change in RES pattern is identified. So corrective redispatch is conducted at the second stage based on contaminated and unspecified RES's PDF  $P_t^w(\alpha)$ . The change in dispatch schedule and total cost with respect to  $P_t^w(\alpha)$  serve as an index revealing the secure and economic risk the system operator may face by adopting the imprecise RESs' prediction.

### A. First-stage model based on $P_t^w(0)$ before contamination

At the first stage, energy and reserve of conventional units are procured simultaneously based on fixed  $P_t^w(0)$  for all  $t$  in dispatch scope  $T$ , where the 0 in brackets means the PDF for wind is uncontaminated. The detailed formulation is as follows:

$$\min \sum_{t=1}^T \sum_{n=1}^{N_g} (c_n^{g(1)} g_{n,t} + \hat{c}_n^{g(1)} \hat{g}_{n,t} + \check{c}_n^{g(1)} \check{g}_{n,t}) + \sum_{t=1}^T \sum_{n=1}^{N_w} c^{curt} \check{w}_{n_w,t} \quad (8)$$

$$\sum_{n_d=1}^{N_d} \bar{d}_{n_d,t} = \sum_{n=1}^{N_g} g_{n,t} + \sum_{n_w=1}^{N_w} (\bar{w}_{n_w,t}(0) - \check{w}_{n_w,t}), \forall t \in T \quad (9)$$

$$\mathbf{f} \mathbf{l}_t(0) = \mathbf{A}^g \mathbf{g}_t - \mathbf{A}^l \mathbf{d}_t + \mathbf{A}^w \bar{\mathbf{w}}_t(0), \forall t \in T \quad (10)$$

$$\mathbf{f} \mathbf{l}^{min} \leq \mathbf{f} \mathbf{l}_t(0) \leq \mathbf{f} \mathbf{l}^{max}, \forall t \in T \quad (11)$$

$$g_{n,t+1} - g_{n,t} + \check{g}_{n,t} + \hat{g}_{n,t+1} \leq r a_n * g_{n,t}, \forall t \in T, \forall n \in N_g \quad (12)$$

$$g_{n,t} - g_{n,t+1} + \hat{g}_{n,t} + \check{g}_{n,t} \leq r a_n * g_{n,t}, \forall t \in T, \forall n \in N_g \quad (13)$$

$$g_n^{min} \leq g_{n,t} \leq g_n^{max}, \forall t \in T, \forall n \in N_g \quad (14)$$

$$g_{n,t} + \hat{g}_{n,t} \leq g_n^{max}, \forall t \in T, \forall n \in N_g \quad (15)$$

$$g_{n,t} - \check{g}_{n,t} \geq g_n^{min}, \forall t \in T, \forall n \in N_g \quad (16)$$

$$L_t^{(1)}(\mathbf{x}^{(1)}, 0) = \sum_{n_d=1}^{N_d} \bar{d}_{n_d,t} - \sum_{n_w=1}^{N_w} (\bar{w}_{n_w,t}(0) - \check{w}_{n_w,t}) - \sum_{n=1}^{N_g} (g_{n,t} + \hat{g}_{n,t}), \forall t \in T \quad (17)$$

$$\left\{ \begin{aligned} v_t^l + \frac{1}{1 - \beta_t^l} \int_{L(x, \tilde{x}) \geq v} [L_t^{(1)}(\mathbf{x}^{(1)}, 0) - v]^+ dP_t^w(0) &\leq R_t^d, \forall t \in T \\ \gamma_{s,t}^d \geq L_t^d - v_t^d, \gamma_{s,t}^d \geq 0, \forall t \in T, \forall s \in S \end{aligned} \right. \quad (18)$$

$$L_t^{w(1)}(\mathbf{x}^{(1)}, 0) = \sum_{n_w=1}^{N_w} (\bar{w}_{n_w,t}(0) - \check{w}_{n_w,t}) - \sum_{n_d=1}^{N_d} \bar{d}_{n_d,t} + \sum_{n=1}^{N_g} (g_{n,t} - \check{g}_{n,t}), \forall t \in T \quad (19)$$

$$\left\{ \begin{aligned} v_t^w + \frac{1}{1 - \beta_t^w} \int_{L(x, \tilde{x}) \geq v} [L_t^{w(1)}(\mathbf{x}, 0) - v]^+ dP_t^w(0) &\leq R_t^w, \forall t \in T \\ \gamma_{s,t}^w \geq L_t^w - v_t^w, \gamma_{s,t}^w \geq 0, \forall t \in T, \forall s \in S \end{aligned} \right. \quad (20)$$

The objective function (8) aims at minimizing the total procurement cost for energy and reserve, while accounting for penalty cost for scheduled wind curtailment. Equation (9) states the active power balance constraint. (10) and (11) are DC power flow limits for avoiding transmission line overloading. Flexibility of thermal units are described in (12) and (13). (14)-(16) define the upper and lower limits for outputs of thermal units. First-stage CVaR for loss of load  $L_t^{(1)}$  and loss of wind  $L_t^{w(1)}$ , i.e., the mean of expected loss of load and expected loss of wind exceeding the confidence levels  $\beta_t^l$  and  $\beta_t^w$ , are stated in (17)-(20), where  $v_t^l, v_t^w, \gamma_{s,t}^d$  and  $\gamma_{s,t}^w$  are auxiliary variables to facilitate CVaR calculation (interested readers can refer to [8] for further details), and 0 in the expressions reveals the wind prediction stay uncontaminated at this stage.

### B. Second-stage corrective model based on $P_t^w(\alpha)$ after contamination

Assume that inaccuracy of wind probability distribution  $P^w(0)$  used at the first stage was identified at the second stage. However, since the first-stage procurement is already completed and implemented, corrective procurement must be run to prevent system from violating security requirements under contaminated PDF  $P^w(\alpha)$ . The formulation for the corrective dispatch is as follows:

$$\min \sum_{t=1}^T \sum_{n=1}^{N_g} (c_n^{g(2)} \Delta g_{n,t} + \hat{c}_n^{g(2)} \Delta \hat{g}_{n,t} + \check{c}_n^{g(2)} \Delta \check{g}_{n,t}) + \sum_{t=1}^T \sum_{n_w=1}^{N_w} c^{curt} \Delta \check{w}_{n_w,t} \quad (21)$$

$$\sum_{n=1}^{N_l} \bar{d}_{n_d,t} = \sum_{n=1}^{N_g} (g_{n,t} + \Delta g_{n,t}) + \sum_{n=1}^{N_w} (\bar{w}_{n_w,t}(\alpha) - \check{w}_{n_w,t} - \Delta \check{w}_{n_w,t}), \forall t \in T \quad (22)$$

$$\mathbf{f} \mathbf{l}_t(\alpha) = \mathbf{A}^g (\mathbf{g}_t + \Delta \mathbf{g}_t) - \mathbf{A}^l \mathbf{d}_t + \mathbf{A}^w (\bar{\mathbf{w}}_t(\alpha) - \check{\mathbf{w}}_t - \Delta \check{\mathbf{w}}_t), \forall t \in T \quad (23)$$

$$\mathbf{f} \mathbf{l}^{min} \leq \mathbf{f} \mathbf{l}_t(\alpha) \leq \mathbf{f} \mathbf{l}^{max}, \forall t \in T \quad (24)$$

$$g_{n,t+1} - g_{n,t} + \check{g}_{n,t} + \hat{g}_{n,t+1} + \Delta \check{g}_{n,t} + \Delta \hat{g}_{n,t+1} \leq r a_n * g_{n,t}, \forall t \in T, \forall n \in N_g \quad (25)$$

$$g_{n,t} - g_{n,t+1} + \hat{g}_{n,t} + \check{g}_{n,t} + \Delta \hat{g}_{n,t} + \Delta \check{g}_{n,t} \leq r a_n * g_{n,t}, \forall t \in T, \forall n \in N_g \quad (26)$$

$$g_{n,t} + \hat{g}_{n,t} + \Delta g_{n,t} + \Delta \hat{g}_{n,t} \leq g_n^{max}, \forall t \in T, \forall n \in N_g \quad (27)$$

$$g_{n,t} - \check{g}_{n,t} + \Delta g_{n,t} - \Delta \check{g}_{n,t} \geq g_n^{min}, \forall t \in T, \forall n \in N_g \quad (28)$$

$$L_t^{(2)}(\mathbf{x}^{(2)}, \alpha) = \sum_{n_d=1}^{N_d} \bar{d}_{n_d,t} - \sum_{n_w=1}^{N_w} (\bar{w}_{n_w,t}(\alpha) - \check{w}_{n_w,t} - \Delta \check{w}_{n_w,t}) - \sum_{n=1}^{N_g} (g_{n,t} + \hat{g}_{n,t} + \Delta \hat{g}_{n,t}), \forall t \in T \quad (29)$$

$$\left\{ \begin{aligned} v_t^l + \frac{1}{1 - \beta_t^l} \int_{L(x, \tilde{x}) \geq v} [L_t^{(2)}(\mathbf{x}^{(2)}, \alpha) - v]^+ dP_t^w(\alpha) &\leq R_t^d, \forall t \in T \\ \gamma_{s,t}^d \geq L_t^d - v_t^d, \gamma_{s,t}^d \geq 0, \forall t \in T, \forall s \in S \end{aligned} \right. \quad (30)$$

$$L_t^{w(2)}(\mathbf{x}^{(2)}, \alpha) = \sum_{n_w=1}^{N_w} (\bar{w}_{n_w,t}(\alpha) - \check{w}_{n_w,t} - \Delta \check{w}_{n_w,t}) - \sum_{n_d=1}^{N_d} \bar{d}_{n_d,t} + \sum_{n=1}^{N_g} (g_{n,t} - \check{g}_{n,t} - \Delta \check{g}_{n,t}), \forall t \in T \quad (31)$$

$$\left\{ \begin{aligned} v_t^w + \frac{1}{1 - \beta_t^w} \int_{L(x, \tilde{x}) \geq v} [L_t^{w(2)}(\mathbf{x}^{(2)}, \alpha) - v]^+ dP_t^w(\alpha) &\leq R_t^w, \forall t \in T \\ \gamma_{s,t}^w \geq L_t^w - v_t^w, \gamma_{s,t}^w \geq 0, \forall t \in T, \forall s \in S \end{aligned} \right. \quad (32)$$

The objective function (21) aims at minimizing the cost for corrective actions induced by wind prediction contamination, which consists of the cost for procuring extra energy and reserve capacity, as well as the penalty cost for extra wind curtailment and load shedding. Equation (22) denotes active power rebalance constraint. Constraints (23) and (24) assure the transmission lines will not exceed the limits under changed wind pattern after redispatch. (25)-(28) represent the residual capability of thermal units to involve in energy and reserve redispatch. (29)-(32) state the CVaR constraints at the second stage. It should be highlighted that the loss functions  $L_t^{(2)}$  and  $L_t^{w(2)}$ , along with random wind output  $\bar{w}_{n_w,t}(\alpha)$  and its distribution function  $P_t^w(\alpha)$  are all coupled with contamination degree  $\alpha$ .

As the contaminated PDF  $P^w(\alpha)$  is assumed to be unspecified at this stage, the optimal solution and minimum redispatch cost for this corrective model are highly dependent on the value of contamination degree  $\alpha$ . Therefore, by perturbing  $\alpha$  continuously from 0 to 1, we are able to use CBT to assess how robust this operating strategy is with respect to RESs' uncertainty.

## IV. CASE STUDY

### A. Test system

In this section, we use a modified IEEE 14-bus system to demonstrate the proposed contamination technique based robustness evaluation method. The detailed topology and network parameters can be found in [9]. Table I shows the parameters for the thermal units, whose total capacity is 400

MW. The energy and reserve price for the second stage are set to be twice as the first-stage values. The tested system has a peak demand of 377.19MW and a minimum demand of 157.92MW in the tested day. Two wind turbines are installed at bus 3 and bus 6, with overall capacity of 220MW, accounting for 35.48% of total generation capacity. Curves for first-stage expected values of wind output and load level in the next 24h are depicted in Fig. 1, with a resolution of 1h.

TABLE I. PARAMETERS FOR THERMAL UNITS

No.	Lower Limit (MW)	Upper Limit (MW)	Energy Price (MW/\$)	Reserve Price (MW/\$)	Ramping Rate (MW/h)	Bus
1	50	200	20	10	30	8
2	10	100	40	20	60	1
3	10	100	60	30	60	2

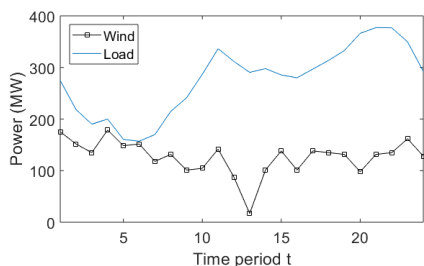


Figure 1. Expected values for wind and load at the first stage

Prediction error for load at both stages and wind at the first stage are assumed to follow Gaussian distribution, with standard deviations of 3% and 10% of their expected values, respectively. As for the CVaR calculations at both stages, confidence levels are set to be 0.95, and the risk tolerance levels are set to be 5% and 20% of expected values of load and wind, respectively. Monte Carlo Simulation is utilized to generate 1000 scenarios for load and wind.

### B. Robustness test

#### 1) Simulation cases

Three wind contamination cases are studied. The first case is denoted by medium wind expectation (MWE), where the probability distribution for wind output is contaminated by a beta distribution  $Q_1$  with same mean value and deviation as first-stage distribution  $P$ . While in the case of high wind expectation (HWE) and the case of low wind expectation (LWE), the PDF for wind output is contaminated by beta distribution  $Q_2$  and  $Q_3$  with same deviation but a higher ( $1.2w_{n,t}$ ) and a lower ( $0.6w_{n,t}$ ) expectation value, respectively, showing an overvalued and undervalued first-stage prediction. Fig. 2 illustrates the three different probability distributions at bus 3 when  $t=3$ .

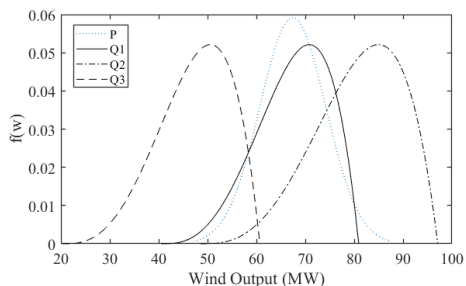


Figure 2. PDFs for wind output at bus 3 in time period 3

## 2) Result

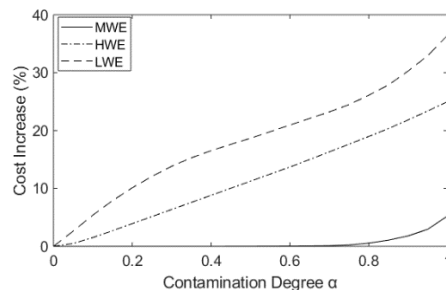


Figure 3. Percentage cost increase

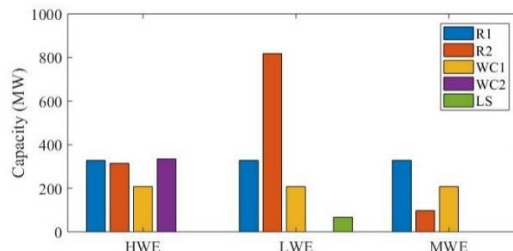


Figure 4. Capacity of procured reserve at first (R1) and second (R2) stage, scheduled wind curtailment at first (WC1) and second stage (WC2) and load shedding (LS) at second stage under 3 cases when  $\alpha = 0.8$

Under different contamination degrees from 0 to 1, the costs of the system change as illustrated in Figure 3, where the percentage cost increase is defined as extra dispatch cost under contamination divided by the cost with perfect prediction. Besides, since the operating schedule would be different as probability pattern of wind output changes, we expect to figure out the detailed influence this change induces. Therefore, Figure 4 is used to illustrate the scheduled capacity of reserve, wind curtailment and load shedding at both stages, where contamination degrees for 3 cases are all set to be 0.8.

It is not surprising that as the contamination progresses, total dispatch costs under all 3 cases increase monotonously with different slope. In case MFE, the cost first remains unchanged within the interval (0, 0.8), which reveals no corrective action is needed. Thus we can say the first-stage decision driven from  $P$  is robust within this interval. However, cost starts to rise as  $\alpha$  reaches 0.8, meaning that corrective actions are needed to ensure operating security. With reference to the orange bar (R2) of Figure 4, we notice that extra reserve capacity is procured at  $\alpha = 0.8$ , as the scheduled reserve at first stage can no longer satisfy the security requirement.

On the other hand, in case HWE and case FWE, the costs both rise evidently at the very start, due to the severe deviation between  $P$  and  $Q_2, Q_3$ . In case HWE, the cost increases nearly linearly, since at high wind penetration level, conventional generations are short of down-reserve capacity due to light-loaded state. Therefore, as contamination aggravates, consecutive wind curtailment is observed as all available down-reserves are already procured and committed at the early phase. The purple bar (WC2) in Fig. 4 can serve as an evidence for this statement. In case LWE, the cost rises even more drastically than HWE. In the forefront of the curve where  $P$  is mildly contaminated by  $Q_3$ , no power supply for end-users would be interrupted, since energy and up-reserve capacity provided by thermal units are relatively sufficient to fill up the gap caused by imprecise prediction. Nevertheless, owing to the restriction of system flexibility, large-scale load shedding occurs at  $\alpha =$

0.8, as shown in green bar (LS) in Figure 4, which also explains the phase transition in the latter part of the curve.

By comparing the results from tree cases, we can conclude that the dispatch strategy is relatively robust in case MPE, where only the shape of wind PDF is imprecise, while reveals a lower resistivity against the contamination case of LPE and HPE. Especially the case LPE, where wind output is overvalued at the first stage, will lead to more undesirable outcomes like higher operational cost and risk of load shedding, due to the scarcity of down-reserve sources in power system with large wind penetration.

### C. Sensitivity Analysis

The objective of this part is to identify the factors that influence the robustness of strategy against contamination. Wind penetration level and ramping rate are chosen to illustrate their contributions. Both tests are conducted under case LPE, which was identified as the severest case from the above analysis, and contamination degree is set to be 0.5.

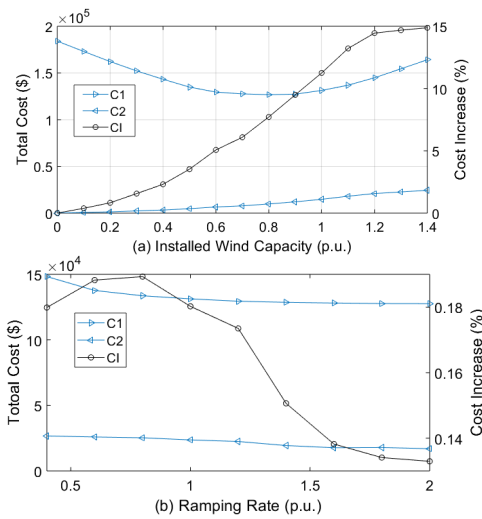


Figure 5. Total dispatch cost at the first (C1) and second (C2) stage, and percentage cost increase (CI) with different installed wind capacity and ramping rate

#### 1) Wind Penetration Level

First, we change the installed wind capacity from 0 to 1.4 times of their original capacity. The change in total dispatch cost is shown in Figure 5(a). It is not surprising that the first-stage cost (C1) drops down at the beginning with the increased wind capacity. Then the curve rises when the installed wind capacity reaches 0.8 p.u., for higher need in reserve to accommodate increasing uncertainty offsets economic benefits wind brings. On the other hand, the remedy cost at the second stage (C2) rises monotonically, as higher wind penetration results in greater gap in energy and reserve between two stages. The percentage change (CI) also increases monotonically, revealing that as more wind generations are integrated, power system may face greater risk of financial loss and security constraints violations.

#### 2) Ramping rate

Flexibility of power systems is determined by the reserve capacity and its responsive rate. Here we investigate the effect of ramping rate on the robustness of strategy. In a similar vein, we perturb the ramping rates of conventional units from 0.4 to

2 times of their original values. It is shown in Figure 5(b) that as the ramping rate increases, first-stage dispatch cost and second-stage corrective cost both drop accordingly, because more reserves from cost-effective units are available. The overall trend of percentage cost change is also in a descending manner, while the trivial increase at the beginning can be interpreted as C1 has slightly faster descending rate than C2 when units have limited ramping capability. Therefore, we still can conclude that improving system flexibility can enhance the robustness of dispatch strategy against contamination.

## V. CONCLUSION

In this paper, a contamination-based technique (CBT) for assessing robustness of power system operating strategy against uncertainty is proposed. Without loss of generality, a contamination-based two-stage stochastic LAD model with inaccurate first-stage prediction is constructed to demonstrate the validity of this evaluation method. The result shows that in a power system with high wind penetration level, the tested stochastic model has comparably high robustness to the contamination where only the shape of RESs' PDFs are imprecise, but has a weak resistance to the contamination with overvalued and undervalued first-stage predictions. Particularly, the contamination with overvalued prediction will induce severer outcomes such as higher dispatch cost and load shedding, due to the lack of flexible sources. Furthermore, the sensitivity analysis reveals that despite higher renewable penetration can provide economic profits considerably, it also worsens the robustness of the operating strategy, which can be enhanced by promoting the system flexibility on the contrary.

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