Two-Stage Stochastic Unit Commitment Considering Distribution Perturbation via Contamination Technique

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***Abstract*—Stochastic unit commitment (SUC) is deemed as an efficient operational strategy to tackle with the uncertainty in power system. However, in order to implement SUC, a predetermined probability distribution function (PDF) *P* or finite scenarios for uncertain parameters is often required, whose true pattern is hard to obtain in real world due to various misspecification. In this paper, contamination technique, by which continuous perturbations on *P* can be achieved, is introduced to study the stability and robustness of SUC with respect to *P*. We show that under certain conditions, analytical upper and lower bounds, i.e., contamination bounds for SUC can be constructed globally. Then, the proposed method is combined with a risk-based two-stage SUC, where wind penetration and demand response are considered. Numerical experiment on a modified IEEE 14-bus system is performed to test the feasibility and efficiency.**

***Index Terms*—contamination technique, stochastic unit commitment, probability distribution**

NOMENCLATURE

A. Indices

i	Index of thermal units
k	Index of DR units
r	Index of wind farms
t	Index of times
s	Index of wind output scenarios
n	Index of buses
l	Index of transmission lines

B. Sets

Ω_n^G	Set of thermal units at bus n
Ω_n^D	Set of DR units at bus n
Ω_n^W	Set of wind farms at bus n

C. Parameters

$L_{n,t}$	Power demand of Load
$P_i^{G,\min}, P_i^{G,\max}$	Minimum/maximum output of thermal unit
T_i^{on}, T_i^{off}	Minimum continuous on/off time of thermal unit
V_i	Ramping rate of thermal unit
$\bar{p}_{n,t}^w$	Forecasted output of wind farm
$p_{n,t,s}^{w,\max}, p_{n,t,s}^{w,\min}$	Maximum/minimum admissible wind output
$\hat{D}_{k,t}, \breve{D}_{k,t}$	Maximum upward/downward reserve offered by DR units
C^G	Energy cost of thermal unit ($/MW)
$\hat{C}^G, \breve{C}^G$	Upward/downward reserve cost of thermal unit ($/MW)
C^{SU}	Start-up cost of thermal unit ($/MW)
$\hat{C}^D, \breve{C}^D$	Upward/downward reserve cost of DR unit ($/MW)
C^{Curt}, C^{Shed}	Cost of wind curtailment/load shedding ($/MW)
$fl_{l,t}^B$	Power flow on transmission line with perfect wind forecast
$fl_{l,t,s}^S$	Power flow on transmission line in scenario s
$fl_l^{\max}$	Upper bound of power flow on transmission line
A	l-colum of power distribution factor matrix

D. Decision Variables and Random Variables

$u_{i,t}$	On/off state of thermal unit
$x_{i,t}, y_{i,t}$	Start-up/shut-down action of thermal unit
$p_{i,t}^G$	Output of thermal unit
$r_{i,t,s}$	Reserve capacity of thermal unit in scenario s
$\hat{r}_{i,t}, \breve{r}_{i,t}$	Procured upward/downward reserve capacity of thermal unit
$d_{k,t,s}$	Reserve capacity of DR unit in scenario s
$\hat{d}_{k,t}, \breve{d}_{k,t}$	Procured upward/downward reserve capacity of DR units

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$a_{i,t}^{G}$	Participation factor of thermal unit
$a_{k,t}^{D}$	Participation factor of DR unit
$\tilde{p}_{i,t}^{W}$	Random output of wind farm
$\tilde{p}_{l,t}^{N}$	Random line rating of transmission line
$\tilde{r}_{i,t}$	Random adjustment capacity of thermal unit
$\tilde{d}_{k,t}$	Random adjustment capacity of thermal unit

I. INTRODUCTION

In power system operation, unit commitment (UC), which decides the optimal schedule and output level for generation in a look-ahead manner, plays one of the most essential roles. With increasing penetration of variable and uncertain resources, e.g. wind and solar, the conventional deterministic UC model can no longer fulfill the operational requirements and guarantee a satisfactory tradeoff between reliability and cost-efficiency. Stochastic unit commitment (SUC) is put forward in this context. SUC models are often constructed as a two-stage [1] or multi-stage problem [2], where generation schedule is decided at the first stage with here-and-now information and power dispatch is then optimized in later stages based on updated information. Besides, risk-based formulation is also prevalent in current literatures [3], where various indices, such as loss of load probability (LOLP) and expected unserved load (EUE) are used to assess the operating risk.

In most cases of SUC, a predetermined probability distribution function (PDF) or scenarios for uncertain parameters is required [3][4]. However, in real-life situations, either for simplification or misspecification, the quality and reliability of given PDFs or scenarios are questionable and hard to quantify. Consequently, the obtained schedule may turn out to be suboptimal, resulting in cost inefficiency and even induce security issues if huge discrepancy between the estimation and realization occurs. In order to address this issue, some efforts have been made by adopting different methodology and perspectives. In [5], the authors investigated how wrong parameters, i.e., inaccurate mean, kurtosis and/or skewness of wind PDF impact the unit commitment (UC) schedule and final operation cost. But this test is conducted purely numerically where few analytical insights are given. Distributionally robust model, where the final decision is obtained by minimizing the objective under worst-case distribution within an ambiguity set, is also under investigation and has been adopted in OPF [6] and UC [7]. However, the formulations in this line tend to be complicated and often requires much computational efforts. Qualitative and quantative stability of stochastic programming with respect to PDF is also tested outside the power system domain[8][9].

Contamination technique is first put forward in [10] to assess the robustness of a basic stochastic formulation. Through contaminating the given distribution for random parameters with another one, the original highly-complicated problem is reduced to a linearly perturbed formulation, which is more tractable. Hereafter, the contamination technique was further utilized to construct the contamination bounds for multistage model [11], risk-constrained model [12], stochastic dominance constrained model [13], and financial problems are used to check its feasibility. In this paper, in order to offer some analytical insights while attaining computational tractability, we try to combine this idea with SUC.

Therefore, the main outcomes and contributions of this paper can be summarized as below:

1) Contamination technique is first introduced to construct the contamination bounds, i.e., the upper and lower bounds for SUC under distribution perturbation. It is proved that under certain conditions, the function of optimal value for SUC is concave with respect to contamination degree. Then, a trade-off strategy is derived via the obtained contamination bounds, by only performing 2 extra runs;
2) A risk-based two-stage SUC formulation with embedded contamination technique is constructed, where CVaR of transmission line overloading is utilized as the risk index. Demand response is also considered to offer services of frequency regulation;
3) Case study is performed on a modified IEEE 14-bus system to show the feasibility of contamination bounds. The performance of the trade-off strategy derived from contamination bounds is also examined, in comparison with other two feasible strategies.

II. CONTRAMINATION TECHNIQUE

A. Basic Concept

The main idea and formulation of contamination technique are illustrated in this subsection. We first assume P which is estimated from finite historical samples cannot reflect the real pattern of random variable $\tilde{\varepsilon}$ precisely due to various reasons such as poor approximation. By continuously perturbing P with another fixed probability distribution Q, which is also obtained through fitting another set of historical data, the contamination process is written as follows:

$$P_{\lambda} = \lambda P + (1-\lambda)Q \qquad \lambda \in [0,1] \tag{1}$$

where λ denotes the contamination degree, i.e., when $\lambda = 0$, the P remains uncontaminated and pure, and random vector $\tilde{\varepsilon}$ perfectly follows the distribution P; when $\lambda = 1$, P is totally contaminated and random vector $\tilde{\varepsilon}$ perfectly follows distribution Q; when $0 \le \lambda \le 1$, P is contaminated by Q partially with certain degree λ.

Then, we consider a two-stage stochastic program based on stated P_{λ}:

$$\hat{\varphi}(P_{\lambda}) = \min_{\boldsymbol{x} \in \boldsymbol{X}(P_{\lambda})} \hat{F}(\boldsymbol{x}, P_{\lambda}) = \min_{\boldsymbol{x} \in \boldsymbol{X}(P_{\lambda})} f_1(\boldsymbol{x}) + \mathbb{E}_{P_{\lambda}}\left[\hat{f}_2(\boldsymbol{x}, P_{\lambda})\right] \tag{2a}$$

$$\boldsymbol{X}(P_{\lambda}) = \boldsymbol{X} \cap \{\boldsymbol{x} \mid g_i(\boldsymbol{x}, \lambda) \le 0\} \tag{2b}$$

where $\boldsymbol{x}$ denotes the vector of decision variables, which may contain both continuous or binary variable. $\boldsymbol{X}$ is a convex set for $\boldsymbol{x}$ which is independent of P. Function $\hat{\varphi}(P_{\lambda})$, $f_1(\boldsymbol{x})$, $\hat{f}_2(\boldsymbol{x}, P_{\lambda})$ and $g_i(\boldsymbol{x}, P_{\lambda})$ $(i = 1,...I)$ denote optimal value, first- and second-stage cost function and constraints, respectively, all of which are dependent on P_λ. $\mathbb{E}_{P_\lambda}[\bullet]$ is the expectation operator under P_λ.

Therefore, by setting different value of λ, we can evaluate the impact of adopting the distribution with different contamination degree.

B. Contamination Bounds and Trade-off Strategy

In order to construct the contamination bounds, i.e., analytical upper and lower bounds for (2), we rewrite (2) into following formation:

$$\varphi(P_\lambda)=\min_{\boldsymbol{x}\in \boldsymbol{X}} F(\boldsymbol{x},P_\lambda)=\min f_1(\boldsymbol{x})+\int_\Omega f_2\left(x,\tilde{\varepsilon}\right)P_\lambda(d\tilde{\varepsilon}) \tag{3}$$

where Ω is the sample space for $\tilde{\varepsilon}$. Note that here we assume $\boldsymbol{x}\in\boldsymbol{X}$, which means the decision variables are extracted from a convex set that is not dependent on P_λ. This is a reasonable setting, since we can add the constraints $g_i(\boldsymbol{x},\lambda)$ into the objective function by multiplying them with penalty costs.

Theorem 1: If $F(\boldsymbol{x},P_\lambda)$ is linear or concave in λ, $\varphi(P_\lambda)$ which represents the optimal value for (3) will be concave with respect to λ.

Proof: As $\varphi(P_\lambda)$ is linear or concave with respect to λ, we can calculate its derivative at $\lambda=\lambda_1,\lambda_2$:

$$\left.\frac{d\varphi(P_\lambda)}{d\lambda}\right|_{\lambda=\lambda_1}=\frac{dF(\boldsymbol{x}^*(\lambda_1),P_\lambda)}{d\lambda}=\frac{dF(\boldsymbol{x}^*(\lambda_1),\lambda P+(1-\lambda)Q)}{d\lambda} \tag{4a}$$
$$\geq F(\boldsymbol{x}^*(\lambda_1),Q)-F(\boldsymbol{x}^*(\lambda_1),P)$$

$$\left.\frac{d\varphi(P_\lambda)}{d\lambda}\right|_{\lambda=\lambda_2}\geq F(\boldsymbol{x}^*(\lambda_2),Q)-F(\boldsymbol{x}^*(\lambda_2),P) \tag{4b}$$

where $\boldsymbol{x}^*(\lambda_1)$, $\boldsymbol{x}^*(\lambda_2)$ denote the optimal solutions at $\lambda=\lambda_1,\lambda_2$. Then, the following inequality holds for any $\lambda_1,\lambda_2\in[0,1]$:

$$\varphi_2(\lambda_1)+\frac{d\varphi_2(\lambda_1)}{d\lambda_1}(\lambda_2-\lambda_1)\geq\lambda_2 F(\boldsymbol{x}_2^*(\lambda_1),Q)+(1-\lambda_2)F(\boldsymbol{x}_2^*(\lambda_1),P)$$
$$\geq\lambda_2 F(\boldsymbol{x}^*(\lambda_2),Q)+(1-\lambda_2)F(\boldsymbol{x}^*(\lambda_2),P)=\varphi_2(\lambda_2) \tag{5}$$

which corresponds to the definition of concave function.□

Therefore, by using the Theorem 1 and formulation (3), the natural and global upper bound UB, together with lower bound LB can be calculated as follows:

$$\text{UB1:}\quad \varphi(P)+\lambda\left.\frac{d\varphi_2(P_\lambda)}{d\lambda}\right|_{\lambda=0}=\varphi(P)+ \tag{6a}$$
$$\int_\Omega f_2\left(\boldsymbol{x}^*(P),\tilde{\varepsilon}\right)Q(d\tilde{\varepsilon})-\int_\Omega f_2\left(\boldsymbol{x}^*(P),\tilde{\varepsilon}\right)P(d\tilde{\varepsilon})$$

$$\text{UB2:}\varphi(Q)+\int_\Omega f_2\left(x^*(Q),\tilde{\varepsilon}\right)Q(d\tilde{\varepsilon})-\int_\Omega f_2\left(x^*(P),\tilde{\varepsilon}\right)P(d\tilde{\varepsilon}) \tag{6b}$$

$$\text{UB=max}\{\text{UB1,UB2}\} \tag{6c}$$

$$LB:\varphi(P)+\lambda\left[\varphi(Q)-\varphi(P)\right] \tag{6d}$$

A simple illustration for the contamination bounds is as shown in Figure 1. Assume UB1 and UB2 intersects at $\lambda=\lambda_h$, we define the decision $\boldsymbol{x}^*(\lambda_h)$ made under $\lambda=\lambda_h$ to be the trade-off strategy, whose performance will be examined in the following sections.

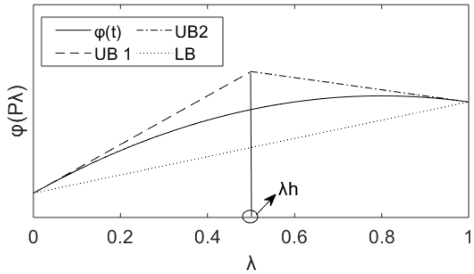


Figure 1. Contamination bounds

III. Mathematical Formulation

A. Assumptions and Simplifications

In order to put our focus on the distribution perturbation, several assumptions are made below for simplicity and clarity:

1) Thermal generations all serve as AGC units to offer frequency regulation following an affine recourse mechanism [3], with their participation factors as decision variables;

2) Load can be precisely predicted, unlike the wind outputs;

3) DR units are scattered among load buses which are controlled integratedly and continuously via load service entities (LSEs). They get involved in frequency regulation, and also treated as AGC units.

B. Two-Stage Stochastic Unit Commitment

1) Objective function

$$\min \sum_{t=1}^{T}\left\{\sum_{i=1}^{N_G}(F_i^G(p_{i,t}^G,\hat{r}_{i,t},\breve{r}_{i,t})+C^{SU}x_{i,t})+\sum_{t=1}^{T}\sum_{k=1}^{N_D}F_k^D(\hat{d}_{k,t},\breve{d}_{k,t})\right.$$
$$+\mathbb{E}_{P_\lambda}[C^W\sum_{i=1}^{N_G}\left(a_{i,t}^G\sum_{r=1}^{N_W}(\bar{p}_{r,t}^W-\tilde{p}_{r,t}^W)-\hat{r}_{i,t}\right)^+$$
$$+C^W\sum_{k=1}^{N_D}\left(a_{i,t}^D\sum_{r=1}^{N_w}(\bar{p}_{r,t}^W-\tilde{p}_{r,t}^W)-\hat{d}_{k,t}\right)^+$$
$$+C^L\sum_{i=1}^{N_G}\left(a_{i,t}^G\sum_{r=1}^{N_W}(\tilde{p}_{r,t}^W-\bar{p}_{r,t}^W)-\breve{r}_{i,t}\right)^+ \tag{7a}$$
$$+C^L\sum_{k=1}^{N_D}\left(a_{i,t}^D\sum_{r=1}^{N_W}(\tilde{p}_{r,t}^W-\bar{p}_{r,t}^W)-\breve{d}_{k,t}\right)^+]$$
$$\left.+C^R\sum_{l=1}^{N_L}\left(v+\frac{1}{1-\beta}\mathbb{E}_{P_\lambda}[\boldsymbol{A}_l\tilde{\boldsymbol{p}}^N-v]^+\right)^+\right\}$$

$$\tilde{p}_{n,t}^N=-L_{n,t}+\sum_{i\in\boldsymbol{\Omega}_n^G}p_{i,t}^G+\sum_{i\in\boldsymbol{\Omega}_n^W}\tilde{p}_{r,t}^W+\sum_{i\in\boldsymbol{\Omega}_n^G}\tilde{r}_{i,t}+\sum_{i\in\boldsymbol{\Omega}_n^D}\tilde{d}_{k,t}\quad\forall n,\forall t \tag{7b}$$

$$\begin{cases}\tilde{r}_{i,t}=-a_{i,t}^G\sum_{r=1}^{N_w}(\tilde{p}_{r,s}^W-\bar{p}_{n,t}^w)\\ \tilde{d}_{k,t}=-a_{i,t}^G\sum_{r=1}^{N_w}(\tilde{p}_{r,s}^W-\bar{p}_{n,t}^w)\end{cases} \tag{7c}$$

where the first line of (7a) indicates the operational cost of thermal units and demand response, consisting of the procurement cost for energy and reserve. Both F_i^G and F_k^D are strictly convex. The second to forth line represent the penalty cost for load shedding and wind curtailment, where $[\cdot]$=max$\{\cdot,0\}$. The last term denotes penalty cost for line overloading, which is measured with the help of CVaR (interested readers can refer to [14] for more details). Here we abuse the notation of P_λ to represent the PDF for the wind output $\tilde{\boldsymbol{p}}^W$. (7c) states the affine relationship between the real-time adjustment of thermal and DR units with the wind deviation.

2) System constraints

$$\sum_{i=1}^{N_G} p_{i,t}^G + \overline{p}_t^w = \sum_{n=1}^{N_B} L_{n,t} \tag{8a}$$

$$\sum_{i=1}^{N^G} a_{i,t}^G + \sum_{k=1}^{N^D} a_{k,t}^D = 1 \tag{8b}$$

(8a) requires that the system should be balanced under expected wind output. (8b) guarantees that the deviation of wind output can be fully tackled.

3) Constraints for Thermal Units and DR units:

$$\begin{cases} x_{i,t} \ge u_{i,t} - u_{i,t-1}, x_{i,t} \ge 0 \\ y_{i,t} \ge u_{i,t-1} - u_{i,t}, y_{i,t} \ge 0 \end{cases} \forall t, \forall i \tag{9a}$$

$$\begin{cases} \sum_{r=t}^{t+T_i^{on}-1} u_{i,r} \ge T_i^{on} x_{i,t} \\ \sum_{r=t}^{t+T_i^{off}-1} (1-u_{i,r}) \ge T_i^{off} y_{i,t} \end{cases} \forall t, \forall i \tag{9b}$$

$$u_{i,t} \cdot P_i^{G,\min} \le p_{i,t}^G \le u_{i,t} \cdot P_i^{G,\max} \quad \forall t, \forall i \tag{9c}$$

$$\begin{cases} u_{i,t} P_i^{G,\min} \le p_{i,t}^G + \hat{r}_{i,t} \le u_{i,t} P_i^{G,\max}, \hat{r}_{i,t} \ge 0 \\ u_{i,t} P_i^{G,\min} \le p_{i,t}^G - \breve{r}_{i,t} \le u_{i,t} P_i^{G,\max}, \breve{r}_{i,t} \ge 0 \end{cases} \forall t, \forall i \tag{9d}$$

$$\begin{cases} p_{i,t+1}^G - p_{i,t}^G + \hat{r}_{i,t+1} + \breve{r}_{i,t} \le V_i u_{i,t} + P_i^{G,\max}(1-u_{i,t}) \\ p_{i,t}^G - p_{i,t+1}^G + \hat{r}_{i,t} + \breve{r}_{i,t+1} \le V_i u_{i,t+1} + P_i^{G,\max}(1-u_{i,t+1}) \end{cases} \forall t, \forall i \tag{9e}$$

$$\begin{cases} \hat{d}_{k,t} \le \hat{D}_{k,t} \\ \breve{d}_{k,t} \le \breve{D}_{k,t} \end{cases} \forall t, \forall k \tag{9f}$$

$$\begin{cases} \sum_{t=1}^{T} \hat{d}_{k,t} \le \hat{D}_k^{\max} \\ \sum_{t=1}^{T} \breve{d}_{k,t} \le \breve{D}_k^{\max} \end{cases} \forall k \tag{9g}$$

(9a) and (9b) force the thermal units to satisfy the minimum continuous on-state and off-state time. Constraint (9c) indicates that outputs of thermal units should be within their technical upper and lower bounds. (9d) set the limits for reserve capacity. (9e) states the ramping capabilities of thermal units, which are intertemporal constraints. Constraints (9f) and (9g) state the maximum reserve the DR units can provide for a single time interval and for the entire dispatch period, respectively.

Therefore, the objective function (7), together with constraints (8), (9) comprise the risk-based two-stage SUC model. As the objective function of the model contains mathematical expectation, it can be conveniently transformed into mix-integer linear programming (MILP), by using Monte Carlo simulation (MCS). Furthermore, $\lambda \neq 0$ the proposed technique only involves the perturbation toward parameter λ, thus will not change the formulation type. Therefore, an investigation can be conveniently conducted with the aid of multiple off-the-shelf MILP solvers. In next section, the optimal value's properties of proposed model with respect to P_λ is tested, by utilizing the above-stated contamination technique to construct the contamination bounds.

IV. Case Study

A. System Description

A modified IEEE 14-bus system is utilized to examine the contamination bounds. The models are coded in CVX 2.1 in MATLAB R2019a and solved by the solver Gurobi 7.58. 8 hours are chosen as the dispatch scope with a time resolution of 1 hour. The detailed topology and network parameters can be found in [11]. Three thermal units are installed with total capacity of 400 MW. A wind farm is installed at bus 3, with capacity of 120MW, accounting for 23.8% of total generation capacity. The tested system has a peak demand of 427.19MW and a minimum demand of 197.92MW in the tested scope. Two DR units are located at bus 6 and 14, with maximum controllable capacity equal to 30% of forecasted demand values at their located buses. Forecasted values of wind output under *P* and load are depicted in Figure 2.

Probability distribution of wind power is assumed to follow Beta distribution *P* with standard deviations of 15% of its expected value. Suppose the distribution of wind power output is perturbed by another Beta distribution *Q,* whose expectation is always 20% larger than *P*, and a standard deviation of 18% of its expected values. Figure 3 shows the *P, Q* at *t*=3. The capacities of transmission lines are all set to be 200 MW. MCS is utilized to generate 1000 scenarios for wind output under assigned P_λ .

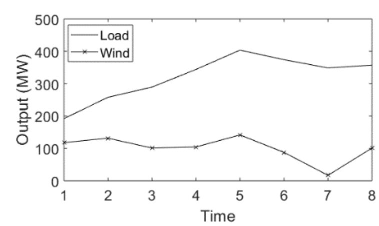


Figure 2. Expected value of wind output and load.

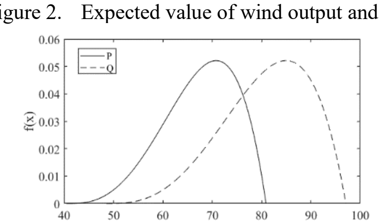


Figure 3. Probability density function of wind power in time period 3.

B. Result

First, the contamination bounds made through contamination technique is validated. By combining the two-stage SUC with (6), we calculate the slopes for UB1, UB2 and LB are 15839.018, -284388 and -12399, respectively, which is shown in Figure 4. We also note that the true optimal value $\varphi(P_\lambda)$, which is depicted as continuous concave function in the figure, is within the constructed bounds.

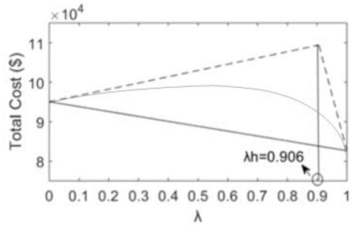


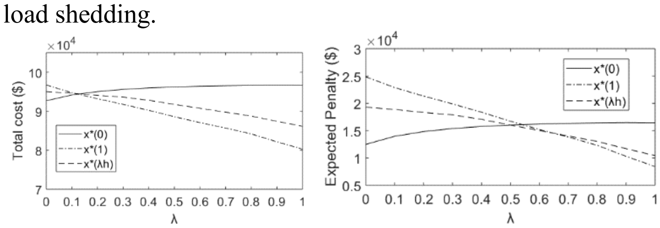
Figure 4. Contamination bounds of the two-stage UC.

Then, three SUC strategies corresponding to different λ are studied. In strategy I and strategy II, the SUC decisions are made assuming that the wind power output follows P distribution and Q distribution, i.e., using decision $\boldsymbol{x}^*(0)$ and $\boldsymbol{x}^*(1)$ made at first-stage respectively. While in strategy III, the $\boldsymbol{x}^*(\lambda_h)$ is made under P_{λ_h} at $\lambda = \lambda_h$ where the two upper bounds intersect, as is shown in figure 4. The contamination bounds and costs under three strategies are presented in figure 4 and Table I.

Table 1 Costs of the two-stage UC under three strategies

	Strategy I	Strategy II	Strategy III
Total Cost ($)	95055	82655	88571
Dispatch Cost ($)	80193	73929	76223
Expected Penalty ($)	14862	8726	12348

In Figure 4, the slope of the objective function at $\lambda = 0$ and λ=1 are 15839.018 and -284388.660. The slope of lower bound is -12399.930. The λ_h is then calculated to be 0.906 and the wind power output used in strategy III is 83.59 MW accordingly. From Table I, we can see that the dispatch cost of strategy III is less than that of strategy I while larger than that of strategy II. This is because with λ equaling to 0, 1 and 0.906, the mean value of wind power output in the three strategies at t=3 are 70 MW, 85 MW and 83.59 MW. Larger percentage wind power integration reveals lower dispatch cost at first stage due to free cost of wind generation. However, when it comes to penalty cost, situation turns out to be different, as SUC with larger λ results in lower penalty. This is because the unit cost of wind spillage is set less than that of load shedding.



(a) (b)

Figure 5. (a) Total costs with different realized λ. (b) Expected penalty with different realized λ.

The performance of three strategies under different realized λ is also studies, and the results are presented in Figure 5. According to figure 5, compared with strategy I and strategy II, the cost of strategy III, i.e., trade-off strategy, remains the medium one however the perturbation changes. Furthermore, under strategy II and III, as the λ increases, the total costs and penalty costs are reduced due to increased wind penetration. However, the total cost and penalty of strategy I present an opposite trend, as they both increase with the rising λ. That is because the wind curtailment have to be conducted at the second stage under the first-stage decision $\boldsymbol{x}^*(0)$ if $\lambda \neq 0$.

V. Conclusion

Misspecification of distribution parameters of stochastic models happens in realistic power system operation. In this paper, contamination technique is used to study the robustness performance of stochastic unit commitment under this context of misspecification. By utilizing the contamination technique, concavity of the proposed two-stage SUC model with respect to contamination degree is proved, which is then utilized to construct the global contamination bounds. A trade-off strategy is subsequently derived based on the bounds. Case study results demonstrate the feasibility and concavity of the contamination bounds on a proposed two-stage SUC model and the superiority of the trade-off strategy. Furthermore, it is also indicated that the less perturbation degree, the less total operation cost. Consequently, promoting the techniques for distribution elicitation, as well as parameter fitting, are essential to hedge against unexpected monetary loss and security issues, which need further investigation and discussion for promoting the future renewable integrations.

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